SPRING 2021: MATH 147 QUIZ 3 SOLUTIONS

Each question is worth 5 points. You must justify your answer to receive full credit.

1. Find and classify the critical points of $f(x, y) = x^2 - 3xy + 5x - 2y + 6y^2 + 8$.

Solution.. Taking partial derivatives and setting them equal to zero, we get the system of equations

$$f_x = 2x - 3y + 5 = 0$$
$$f_y = -3x + 12y - 2 = 0$$

Multiplying the first equation by 3 and the second equation by 2 gives

$$6x - 9y = -15$$
$$-6x + 24y = 4$$

Adding the equations and solving for y yields $y = -\frac{11}{15}$. Substituting this into the first equation above gives $x = -\frac{54}{15}$. Thus, the only critical point is $(-\frac{54}{15}, -\frac{11}{15})$. To test this point, we have $f_{xx} = 2, f_{yy} = 12$ and $f_{xy} = -3$. Thus, $D(-\frac{54}{15}, -\frac{11}{15}) = (2)(13) - (-3)^2 > 0$. Since $f_{xx}(-\frac{54}{15}, -\frac{11}{15}) > 0$, we have a relative minimum at the critical point $(-\frac{54}{15}, -\frac{11}{15})$.

2. Find and classify the critical points of $f(x, y) = x^2 + y^3 - 3y + 1$.

Solution. Taking partial derivatives and setting them equal to zero we get the system of equations

$$f_x = 2x = 0$$
$$f_y = 3y^2 - 3 = 0.$$

The solutions to this system are $x = 0$, $y = \pm 1$, so that we have two critical points (0,1), (0, -1).

For (0,1): $f_{xx}(0, 1) = 2$, $f_{yy}(0, 1) = 6$, $f_{xy}(0, 1) = 0$, Therefore, $D(0, 1) = 2 \cdot 6 - 0^2 > 0$. Thus, $f(x, y)$ has a relative minimum at (0,1).

For (0,-1): $f_{xx}(0, -1) = 2$, $f_{yy}(0, 1) = -6$, $f_{xy}(0, 1) = 0$, Therefore, $D(0, -1) = 2 \cdot (-6) - 0^2 < 0$. Thus, $f(x, y)$ has a saddle point at (0,-1).